

# Bargaining, Competition and Efficient Investment

Kalyan Chatterjee<sup>1</sup>  
Department of Economics,  
The Pennsylvania State University,  
University Park, Pa. 16802, USA

Y. Stephen Chiu  
School of Economics and Finance  
University of Hong Kong,  
Hong Kong

March 23, 2005.

<sup>1</sup>We thank Hongbin Cai, George Mailath, Abhinay Muthoo and Andrew Postlewaite for useful comments.

### **Abstract**

This paper explores the interplay between choice of investment type (specific vs. general), bargaining extensive form and endogenous outside options in the framework of incomplete contracts introduced formally in the work of Grossman, Hart and Moore. We find that the bargaining procedure chosen has significant implications for choice of investment and for the usefulness of the assignment of property rights in enhancing efficiency. Somewhat paradoxically an “auction-like” procedure might need the correct assignment of property rights while a sequential offers procedure might do as well as the best assignment of property rights.

# 1 Introduction

## 1.1 Main features

The aim of this paper is to investigate investment choice by agents in a “thin” market where transactions take place by bilateral bargaining. We study the simplest non-trivial model of this type with two buyers and one seller. Our model has the following features.

1. Both seller and buyers potentially invest. The seller chooses both the type of the investment and the level. The type of the investment varies continuously from 0 (investment specific to the first buyer’s product) to 1 (investment specific to the second buyer’s product) with a value of  $1/2$  being the most “generalist” type. Investment specific to one buyer creates the most value in a transaction with that buyer per unit of investment chosen and the least value in a transaction with the other buyer. We assume the seller’s investment is “more important” than the buyers’ in a sense to be explained later.
2. The environment is one of incomplete contracting. Neither investment level or type or value can be contracted on. Each agent owns an asset in the benchmark model and the investment is in learning how to use the asset for a specific purpose, either for one specific buyer or for something generalist. (The investment is “inalienable”).
3. The “outside option” of the seller in a negotiation with one buyer is the value of an agreement with the other buyer. (The alternative to a negotiated agreement for a buyer is an inside option, that is he produces the item himself.) Since the bargaining institution affects not only the payoff within a negotiation but also the outside option, the nature of the investment chosen depends crucially on it. We consider two extensive

forms, which we think of as corresponding to different kinds of market (as in Shaked (1994) though our markets are a bit different from his). In a Bertrand-type “bazaar”, the short side of the market (here the seller) obtains all the surplus while the long side gets nothing. This is also in the core of the bargaining game. Since the seller’s investment is more important (see Section 4.2), this has the consequence of providing appropriate incentives for the seller to choose the efficient *level* of investment. However, this need not lead to efficiency. In the second market, probably more relevant to modern industry, a transaction involves a single buyer and seller exchanging offers in any one period (like a merger negotiation), and the short side does not get the entire surplus. Paradoxically, this could turn out to be more efficient in terms of choice of investment.

4. The most important feature of our model is the interplay between investment choice and bargaining. Bargaining is sometimes modelled in literature of this kind as an arbitrary division of the total surplus in negotiation and renegotiation and the consequences for investment choice are investigated. In this paper, the extensive form is fixed but the share of surplus is endogenously determined by the investment itself. It is not possible here to consider these two features separately.
5. The efficient assignment of property rights turns out to depend on the type of market we are considering. If it is a “bazaar”, assigning property rights to a buyer for the asset of the seller in addition to her own asset might improve efficiency by ensuring the right choice of type of investment. For the other case, it makes no difference.

In order to fix our minds on the nature of the phenomena being studied, consider a firm of marketing consultants who can invest in specialised knowledge

about one industry (say biotechnology) or invest in skills that are as relevant for diet soft drinks as for the biotechnology industry. The type of investment and the amount of investment both matter here. A similar idea in the academic labour market is mentioned in Chatterjee and Marshall (2003); an academic who chooses to invest in her own speciality will become more valuable in that speciality but will lose a potential job opportunity in, say, a consulting firm.

## 1.2 Related literature

The paper most closely related to this is our own earlier work on investment and competition (Chatterjee and Chiu (2000)). That paper essentially dealt with the choice of type of investment and considered the “bazaar-like” extensive form leading to a core allocation, showing that competition could increase inefficiency. In that paper, only one side of the market had to choose investment. Further, the interplay between level and type of investment and the bargaining procedure was not considered in the earlier (unpublished) paper, though the single bargaining procedure considered had  $m$  sellers and  $n$  buyers, with  $n > m \geq 1$ . Felli and Roberts (2002) and Cole, Mailath and Postlewaite (2001a,b) discuss a similar problem though the latter set of authors analyses the market part by using cooperative game theory<sup>1</sup>. Felli and Roberts do use a Bertrand-type mechanism for choosing allocations and prices but again only one side of the market invests. Both groups of authors obtain efficiency in any mechanism where the bargaining process leads to an allocation in the core and neither considers the nature of the investment, only its level. De Meza and Lockwood (1998b) analyse a somewhat different search and bargaining model in which sellers do not know a priori which buyer they will meet in the transaction phase. They use this as an explanation

---

<sup>1</sup>As they have pointed out, core allocations can also be obtained through several non-cooperative procedures and the absence of discounting in their paper is a more substantial difference.

why complete contracts cannot be written before players choose investment levels.

The paper by Cai (2003) addresses the issue of specific investment, though in a somewhat different way from this paper. His model has two kinds of investment, specific, in the relationship and general, in the outside option. Thus he chooses a more “reduced form” approach than we do in this paper; his paper has no explicit bargaining and the outside option and its change with general investment are exogenously specified. In our model, there is only one investment for the seller, who must, however, choose how specific she wants it to be. The seller therefore chooses her market power but is constrained by the nature of the market (extensive form) in determining the extent to which this power translates into higher payoffs. In the first extensive form we consider, joint ownership would not in fact be an optimal assignment of property rights; such rights should be assigned to the buyer. (The conclusion therefore also differs from Cai’s paper.)

While these papers are the most relevant for understanding competition and specificity, there is a long list of papers on incomplete contracts and property rights beginning with the seminal work of Grossman and Hart (1986) and Hart and Moore (1990). Gans (2003) has an interesting variation in which property rights are not assigned but assets are sold by auction. Chiu (1998), de Meza and Lockwood (1998a) and Rajan and Zingales (1998) explore how the optimal assignment of property rights in Hart and Moore can be reversed by considering outside options as they appear in the strategic bargaining literature starting from Rubinstein (1982). The relevant bargaining literature on outside and inside options is ably surveyed by Muthoo (1999).

## 2 The Model

There are three agents, one seller ( $S$ ) and two buyers ( $B_1$  and  $B_2$ ). The seller can make one unit of the good or service; each buyer has a demand for at most one unit. The maximum price a buyer  $i$  is willing to pay depends on the value  $v_i$ , which can be enhanced by seller investment in human capital. The buyer can also produce the item himself in which case the value to him will be  $\underline{v}$  ( $= 0$ , for convenience).

We consider two extensive form games. In both of these the seller has to choose  $\gamma$ , the investment in knowledge, and the cost of the investment is given by  $c(\gamma)$ , which is increasing, strictly convex, and differentiable everywhere. Provided that the investment  $\gamma$  is fully specific to buyer  $B_i$ , the value to the buyer  $B_i$  is given by

$$v_i = v_{0i} + \alpha\gamma, \tag{1}$$

where  $\alpha$  is a coefficient and  $v_{0i}$  is a component resulting from investment by buyer  $B_i$ . In this paper, we assume that the buyer investment component is less important than the seller investment component in a sense to be made more precise later on (see Section 4.2).

In case the investment is not fully specific to the buyer, the value is reduced. This is captured by the seller's choice of  $x$ . The specification of the investment type is more fully described in Section 4.1.

Both seller and buyers' investment decisions are made at time 0. Bargaining occurs at time 1.

The extensive forms of the bargaining differ in what happens subsequent to the investment being made. The investment, in common with the usual practice in this literature, is observable but contracts cannot be written ex ante in which the terms differ for different values of investment.

The following extensive forms are considered.

**I. Auction-like procedure** The buyers,  $B_1$  and  $B_2$  make price offers to the seller, who accepts at most one. If an offer is accepted the game ends; if not, the seller makes a counter offer in the next period to one of the buyers who then accepts or rejects and so on. Each time there is a rejection, payoffs are discounted as is usual in bargaining theory.

**II. Sequential procedure** The bargaining in the second extensive form is sequential; in the first period a randomly chosen buyer makes an initial offer to the seller who either accepts the offer or rejects it and proceeds to the next period (an even period) to either make a counter-offer to the same buyer or switch to the other buyer and make an offer. Again payoffs obtained in periods after a rejection are appropriately discounted.

Any acceptance ends the game. All agents have the same discount factor. Note that while there is discounting during bargaining, there is not between time 0 and time 1, at which the bargaining starts.

The two different bargaining procedures correspond to different institutional settings. The first one is most "market-like" or "auction-like" though, unlike in an auction, the game could extend for longer than one period. The interaction between bargaining procedure and the investment decision is at the heart of this paper. The bargaining procedures itself have been studied (without investment choice), for example in Osborne and Rubinstein (1990) and Chatterjee and Dutta (1998).

In the example we discussed briefly in the last section, the second extensive form appears more natural, since a buyer seeking to hire a firm of marketing consultants would presumably want to meet them separately first and then make an offer, rather than calling on the consultants to call out a price for their

services. This implicit choice of extensive form could be due to uncertainty about the quality of the consultants or about hesitation in sharing proprietary information, though we do not model these factors explicitly in this paper.

## 2.1 Payoffs and seller/buyer investment

Suppose seller  $S$  invests  $\gamma$ , obtains a price  $p$  at time  $t$ . Then her payoff is

$$\delta^{t-1}p - c(\gamma),$$

where  $\delta$  is the common discount factor and  $c(\gamma)$ , as mentioned earlier, is the cost of investment.

Buyer  $i$  has a payoff of  $\delta^{t-1}(v_i - p)$  if he purchases from the seller and pays a price  $p$  at time  $t$ , and obtains  $\delta^{t-1}v$  if he chooses to opt out and produce himself at time  $t$ . In Section 4.2, we consider buyer investment in determining  $v_{0i}$ . There  $v_{0i} = \beta a_i$ , where  $a_i$  is the investment made by  $B_i$ , and the cost of investment is again  $c(a_i)$ . In Section 4.2, we spell out exactly how the buyer investment is less important than the seller investment.

## 3 Equilibrium of Extensive Form 1 (the “auction-like” mechanism)

We consider pure strategy subgame perfect equilibria of this game. Therefore, we first obtain the equilibria in the bargaining subgame. Recall this begins with the buyers making simultaneous offers to the seller who can choose to accept or reject.<sup>2</sup>

The values  $v_1$  and  $v_2$  are, of course, endogenously determined by seller investment. We therefore have to consider all possible combinations of these

---

<sup>2</sup>A question that has been asked concerns the order of moves—would it matter if the seller moved first in making offers. Such a game would be identical with our game if the initial seller offer were to be rejected. The first period would therefore have a unique continuation payoff in the event of a rejection and the seller’s payoff would be different only to the extent that discounting makes it preferable to be a proposer rather than a responder.

quantities.

**Case (i)** The outside option  $\underline{v} > v_1, v_2$ . In this (trivial) case, there is no trade and  $B_1, B_2$  take their outside options. We rule this out by assuming  $v_i > 0 = \underline{v}$

**Case (ii)** The maximum price  $B_2$  can offer  $S$ ,  $v_2$ , is lower than the price  $S$  would obtain by bilateral bargaining with  $B_1$  alone (assuming for the moment that  $v_1 > v_2 \geq \underline{v} = 0$ ). That is  $v_2 \leq \frac{\delta v_1}{1+\delta}$ . In this case,  $B_2$  is irrelevant in the post-investment matching and bargaining and  $S$  and  $B_1$  play a bargaining game.

**Case (iii)** Keeping the assumption that  $v_1 \geq v_2$ , assume now that  $v_2 > \frac{\delta v_1}{1+\delta}$ .

**Lemma 1** *In case (iii), the following strategies constitute a subgame perfect equilibrium in the bargaining subgame.*

1.  $B_1$  and  $B_2$  make offers of  $v_2$  whenever it is the buyers' turn to make an offer and both buyers are in the game (neither has exercised his outside option).
2. Suppose both buyers are present. Whenever one or both of the offers from them are at least  $v_2$ , the seller accepts the higher one and the one from  $B_1$  in case of a tie. The seller never accepts any offer below  $v_2$ .
3.  $S$  asks  $B_1$  for a price  $p$  such that  $v_1 - p = \delta(v_1 - v_2)$  and  $B_1$  accepts.  $B_1$  accepts any price less than it and rejected any higher price.
4. If only  $B_i, i = 1, 2$ , is present the strategies followed are identical with the Rubinstein alternating-offers bargaining game.

**Proof.** It is clear that the strategies above constitute a subgame perfect equilibrium in the bargaining subgame. ■

**Remark 1** *It is also clear that the outcome is also the unique subgame perfect equilibrium outcome. If there is only one buyer (the other opts out), the equilibrium payoff is uniquely given by Rubinstein's result. Suppose therefore that both the buyers are present. Any  $p < v_2$  being accepted by the seller cannot be an equilibrium outcome because buyers will bid it up. Suppose there is an equilibrium in which the maximal bargaining payoff of the seller in any equilibrium is  $M_S > v_1 - \delta(v_1 - v_2) > \frac{v_1}{1+\delta}$ .  $B_1$  should reject and offer  $\delta M_S$ ; and this will be accepted and give  $B_1$  a higher payoff since  $v_1(1 - \delta) - M_S(1 - \delta^2) > 0$ . A higher payoff for a buyer can be ruled out in similar fashion.*

## 4 Investment in the “Auction-like” Mechanism

### 4.1 The Seller Investment Decision

We now consider the first stage of the game where players invest to increase the surplus before the bargaining takes place.

In this subsection only the seller invests. In the next subsection we shall consider investment by the buyers.

The seller can choose a type of investment,  $x \in [0, 1]$ , as well as a level of investment  $\gamma$ . The value of the item to  $B_1$  will then be

$$v_{01} + g(1 - x) \bullet \alpha\gamma$$

and to  $B_2$

$$v_{02} + g(x) \bullet \alpha\gamma.$$

Here  $g(\bullet) : [0, 1] \rightarrow [0, 1]$  is an “effectiveness” function. We assume  $g(0) = 0$ ,  $g(1) = 1$ ,  $g$  strictly concave, monotonic and twice differentiable everywhere.

The strict concavity assumption is there to ensure that lack of specificity of investment does not cause "too much" loss in output, since  $g(\frac{1}{2}) > \frac{1}{2}$ .

Thus, if  $x = 0$  or  $1$ , the seller specializes his knowledge acquisition to the needs of a specific buyer, while for any  $x \in (0, 1)$ , the investment is of value to both buyers. The cost of investment  $\gamma$  is given by a strictly convex function  $c(\gamma)$ , while the choice of  $x$  is costless.

We now characterize the optimal choice of investment of the seller.

**Proposition 1** *Suppose  $v_{01}$  and  $v_{02}$  are known and are sufficiently close<sup>3</sup>. Then Player  $S$  will choose  $x$  and  $\gamma$  in such a way that  $v_1 = v_2$ . In particular, when  $v_{01} = v_{02}$ , she will choose  $x = \frac{1}{2}$  and  $\gamma = \gamma^*$  where*

$$c'(\gamma^*) = g\left(\frac{1}{2}\right) \bullet \alpha. \quad (2)$$

**Proof.** Suppose on the contrary  $v_1 > v_2$ . There are two cases.

Case (i)  $v_1 > v_2 > \frac{\delta}{1+\delta}v_1$ . Then the price paid by  $B_1$  is  $v_2$ . By increasing  $x$ ,  $v_2$  will increase for the same value of  $\gamma$ , and therefore the original value of  $x$  could not have been optimal.

Case (ii)  $\frac{\delta}{1+\delta}v_1 \geq v_2$ . Then, from the discussion preceding Lemma 1, the price paid by  $B_1$  is  $\frac{\delta}{1+\delta}v_1$ . To maximize this value, since  $B_2$  here becomes irrelevant to the payoff for  $S$ , the seller's optimal choices of  $x$  and  $\gamma$  must be such that is  $x = 0$  and  $c'(\gamma) = \frac{\delta}{1+\delta} \bullet \alpha$ .  $S$ 's payoff is then

$$\frac{\delta}{1+\delta} [v_{01} + g(1) \bullet \alpha \gamma] - c(\gamma).$$

But for the same value of  $\gamma$ , suppose  $x$  is chosen so as to make  $v'_2 = \frac{v_1}{2}$ <sup>4</sup>, that is

$$v_{02} + g(x) \bullet \alpha \gamma = \frac{1}{2} [v_{01} + \alpha \gamma]. \quad (3)$$

<sup>3</sup>This holds if the buyer investment is "less important" than the seller's; see section 4.2 and the next footnote.)

<sup>4</sup>This is possible if the difference between  $v_{01}$  and  $v_{02}$  is not too great, as is implied by the condition that the buyer investment is less important than the seller investment. If  $\beta a^*$  is the highest possible value of buyer investment (see Section 4.2) then this is possible if there exists a  $x$  such that  $\frac{1}{2}[\beta a^* + \alpha \gamma] = g(x)\alpha \gamma$ . This is certainly satisfied if  $\beta a^* < \alpha \gamma$ , where  $\frac{\delta}{1+\delta}\alpha = c'(\gamma)$ , since some  $x$  less than unity will have the desired effect.

Now  $v'_1 = v_{01} + g(1-x)\alpha\gamma$ , and  $v'_1 - v'_2 = v_{01} - v_{02} + g(1-x)\alpha\gamma - g(x) \bullet \alpha\gamma$ .

Using (3), we have

$$\begin{aligned} v'_1 - v'_2 &= v_{02} + g(x) \bullet \alpha\gamma + g(1-x)\alpha\gamma - g(1)\alpha\gamma, \\ &= v_{02} + \alpha\gamma[g(x) + g(1-x) - g(1)] > 0.^5 \end{aligned}$$

Therefore, for the same value of  $\gamma$ , the seller could obtain a payoff of  $v'_2 > \frac{\delta}{1+\delta}v_1$ . This is contradictory to the claim that  $\frac{\delta}{1+\delta}v_1 \geq v_2$ .

The cases where  $v_2 > v_1$  are symmetric to the ones considered here and similar reasoning leads to the same conclusions as here. This completes the proof that  $v_1 = v_2$ . The specific results for the case of  $v_{01} = v_{02}$  are straightforward and omitted. ■

#### 4.1.1 Comparison with the Single Buyer Case

The presence of a second buyer has two opposing effects on seller investment, compared to the case of a single buyer. With a single buyer,  $S$  invests  $\gamma'$  such that

$$c'(\gamma') = \frac{\delta}{1+\delta} \bullet \alpha,$$

and  $x = 0$  or  $1$ .

With two (symmetric) buyers,  $x = \frac{1}{2}$  and

$$c'(\gamma^*) = g\left(\frac{1}{2}\right) \bullet \alpha,$$

with

$$g\left(\frac{1}{2}\right) > \frac{1}{2} > \frac{\delta}{1+\delta}.$$

Only one buyer can be served by the seller so the social surplus due to the investment is

$$v_0 + \alpha\gamma' - c(\gamma')$$

for the first case and

$$v_0 + g\left(\frac{1}{2}\right) \bullet \alpha \gamma^* - c(\gamma^*)$$

in the second case (To allow for proper comparison, here we assume  $v_{01} = v_{02} = v_0$ ).

The two effects are that: (i)  $g\left(\frac{1}{2}\right) < 1$  and therefore the social surplus is smaller when  $B_1$  and  $B_2$  are both present, for the same fixed amount of investment, and (ii)  $\gamma^* > \gamma'$  because the seller obtains all the benefit from increasing investment with two buyers present.

In general, it is not possible to specify which effect will dominate, since this depends on how close  $g\left(\frac{1}{2}\right)$  is to  $\frac{1}{2}$  and  $\gamma^*$  to  $\gamma'$ .

**Example 1:** Suppose  $c(\gamma) = \frac{1}{2}\gamma^2$  and  $g\left(\frac{1}{2}\right) = \frac{1}{2} + \varepsilon$ , and  $\delta \approx 1$ .

Then

$$\begin{aligned} c'(\gamma^*) &= \gamma^* = g\left(\frac{1}{2}\right) \bullet \alpha = \alpha \left(\frac{1}{2} + \varepsilon\right). \\ c'(\gamma') &= \gamma' = \frac{1}{2} \bullet \alpha \end{aligned}$$

(i) Total surplus with  $B_1$  alone

$$\begin{aligned} &= v_0 + \alpha \bullet \frac{1}{2}\alpha - \frac{1}{2} \frac{1}{4} \bullet \alpha^2. \\ &= v_0 + \frac{3}{8}\alpha^2. \end{aligned}$$

(ii) Total surplus with  $B_1$  and  $B_2$

$$\begin{aligned} &= v_0 + \left(\frac{1}{2} + \varepsilon\right) \bullet \alpha \bullet \alpha \left(\frac{1}{2} + \varepsilon\right) - \frac{1}{2} \left(\left(\frac{1}{2} + \varepsilon\right) \alpha\right)^2 \\ &= v_0 + \alpha^2 \left(\frac{1}{2} + \varepsilon\right)^2 \left(1 - \frac{1}{2}\right) \\ &= v_0 + \alpha^2 \bullet \frac{1}{2} \left(\frac{1}{2} + \varepsilon\right)^2 \end{aligned}$$

When  $\varepsilon$  is close to 0, this is clearly less than the surplus with  $B_1$  alone. If  $\varepsilon = \frac{1}{2}$ , its largest possible value, then the total surplus in (ii) is higher.

**Example 2:** Here the  $c(\bullet)$  does not satisfy strict convexity since we assume it to be linear ( $c(\gamma) = c\gamma$ ) and suppose that  $\gamma$  must be between 0 and 1 (both inclusive).

Again with  $\delta$  close to 1, the payoff to the seller from case (i) is

$$\frac{1}{2} [v_0 + \alpha\gamma] - c\gamma.$$

Obviously,  $\gamma' = 1$  if  $\frac{1}{2}\alpha > c$  and 0 otherwise.

Similarly, from case (ii),  $\gamma^* = 1$  if  $(g(\frac{1}{2}) \bullet \alpha - c) > 0$  and 0 otherwise.

The total surplus will be lower under (ii), except if  $\gamma^* = 1$  and  $\gamma' = 0$ , in which case the addition of the second buyer has a beneficial effect.

There is, therefore, a non-trivial subset of parameter values for which the two buyer/one seller auction-like mechanism reduces the total surplus from the transaction, in the absence of verifiable and enforceable contracts.

## 4.2 Buyer investment with the auction-like mechanism

Suppose now that the quantity  $v_{0i}$  in the expression for the value produced by a seller-buyer transaction is in fact dependent on investment by the buyer concerned. We assume that this investment is done at time 0 at the same that seller's choices of  $\gamma$  and  $x$  are made, before the bargaining begins. Let  $v_{0i} = \beta a_i$ , where  $a_i$  is the amount of investment chosen by buyer  $B_i$ ,  $i = 1, 2$ . Suppose the cost of the investment to  $B_i$  is given by a strictly convex function again, namely  $c(a_i)$ ,  $i = 1, 2$ . The buyers are identical in this respect as well, that they have the same cost functions.

To recall, we are going to assume here that the buyer investment, which affects  $v_{0i}$ , is "less important" than the seller investment in the following sense.

First of all, define  $a_i^*$  so that

$$\beta = c'(a_i^*). \quad (4)$$

This is the largest value of investment that a rational buyer can possibly choose in equilibrium. We impose the following restriction on  $a_i^*$ .

If

$$v = \frac{\delta}{(1+\delta)} \alpha \gamma^*,$$

where

$$c'(\gamma^*) = \frac{\delta}{1+\delta} \bullet \alpha, \quad (5)$$

then

$$v > \beta a_i^* \text{ for } i = 1, 2.$$

Then for the maximum possible value of  $\beta a_i^*$ , the seller can choose a  $x$  to get a higher payoff than she would with investment specific to buyer  $i$ . Note that  $g(\frac{1}{2}) > \frac{1}{2} > \frac{\delta}{1+\delta}$ .

The above condition guarantees that the seller will prefer to have the two buyers compete away their surplus rather than obtaining half of the surplus with the buyer who has invested more.

The above restriction is a simplifying assumption; relaxing it would not change the basic qualitative conclusions of this paper, though it could give rise to asymmetric equilibria in the auction-like mechanism.<sup>6</sup>

**Proposition 2** *Suppose that  $S$  chooses an investment level  $\gamma^*$  (as defined in (5)) and sets  $x = \frac{1}{2}$ . Then there is no symmetric pure strategy equilibrium in which  $a_1 = a_2$ . There is a symmetric mixed strategy equilibrium in which each buyer gets an expected payoff of 0.*

---

<sup>6</sup>Obviously, changing the assumption might change the result, but the main point of the paper is that if buyers are not too dissimilar, the seller will choose type of investment (inefficiently) to induce competition among the buyers, and the success of this strategy depends on the bargaining procedure. This "punch line" is not affected if we make buyers very dissimilar.

**Proof.** (i) Suppose there is a symmetric pure strategy equilibrium with  $a_1 = a_2$ . Then  $a_1 = a_2 \neq 0$ , because otherwise some  $B_i$  would deviate and set  $a_i = a_i^*$ . (The surplus from this additional investment would all go to the buyer.) Suppose that  $a_i^* > a_1 = a_2 > 0$ . This cannot be an equilibrium because some buyer would deviate to  $a_i^*$  and do better. Suppose therefore that  $a_1 = a_2 = a_i^*$ . Since both buyers have the same values, the price to be paid to the seller will be  $\beta a_i^* + g(\frac{1}{2})\alpha\gamma^*$  (i.e., the total surplus from the relationship) so each buyer will get a negative payoff because of the cost of investment. Therefore, some player deviating to 0 investment will obtain a higher profit.

(ii) For any mixed strategy equilibrium (symmetric or not), we first note the lower bounds of the supports for both buyers must be the same, i.e.,  $\underline{a}_1 = \underline{a}_2$ . Otherwise, the one whose lower bound is lower will always make a loss by choosing any positive investment strictly below the other buyer's lower bound (as he will never win to recoup its investment cost), and this buyer could have done better by not investing at all. We next claim that the two lower bounds of supports must indeed be zero, i.e.,  $\underline{a}_1 = \underline{a}_2 = 0$ , and as a consequence the expected payoff of each buyer must be zero. Suppose not so that  $\underline{a}_1 = \underline{a}_2 > 0$ . Then buyer  $i$  choosing  $a_i = \underline{a}_i$  wins with positive probability only if  $a_j = \underline{a}_j = \underline{a}_i$ , where  $j \neq i$ . But the payoff to  $B_i$  from such an outcome is still negative ( $-c(\underline{a}_i)$ ) and  $B_i$  can gain by setting  $\underline{a}_i = 0$ . Finally, to calculate the mixed strategy  $F_i(\bullet)$ , we set up the usual expression for the expected payoff of  $B_j$  for any point  $a_j$  in  $B_j$ 's support, where  $i = 1, 2$ , and  $j \neq i$ . The expected payoff equals

$$\beta \int_0^{a_j} (a_j - a_i) dF_i(a_i) - c(a_j).$$

Differentiating the expression with respect to  $a_j$  and setting the resulting term equal to 0, we obtain

$$\beta F_i(a_j) - c'(a_j) = 0.$$

Therefore

$$F_i(a) = \frac{c'(a)}{\beta}. \quad (6)$$

Since this expression applies to both buyers,  $F_1(\bullet) = F_2(\bullet)$  and the mixed strategy equilibrium is indeed symmetric. ■

**Remark 2** *It can be verified from (5) that if  $c(a_i) = \frac{1}{2}a_i^2$ ,  $F(a_i)$  is uniform from 0 to  $\beta$ .*

We note that, for the seller,  $\gamma^*$  is still the equilibrium strategy, since the payoff to  $S$  is  $E\{\min\{\beta\tilde{a}_i, \beta\tilde{a}_2\}\} + g\left(\frac{1}{2}\right)\alpha\gamma - c(\gamma)$ , given that  $x = \frac{1}{2}$  is still optimal.

Asymmetric equilibria do not exist in this game because of the assumption that buyer investment is less important than the seller's. Consider the following example.

**Example 3:** We assume that  $c(\gamma) = \frac{1}{2}\gamma^2$  as in Example 1. Consider the following candidate (investment) equilibrium strategies:

$B_1$  : Invest  $a'_1$  such that

$$\frac{1}{1+\delta}\beta = a'_1.$$

$B_2$  : Invest 0.

$S$  : Set  $x = 0$  and invest  $\gamma'$  such that

$$\gamma' = \frac{\delta}{1+\delta}\bullet\alpha.$$

Let us take  $\delta \rightarrow 1$  for this example. Then  $S$ 's payoff will be

$$\frac{1}{2}[\beta a'_1 + \alpha\gamma'] - \frac{1}{2}\gamma'^2$$

$$\begin{aligned}
&= \frac{1}{2} \left[ \beta \bullet \frac{1}{2} \beta + \alpha \frac{1}{2} \alpha \right] - \frac{1}{2} \left( \frac{1}{2} \alpha \right)^2 \\
&= \frac{\beta^2}{4} + \frac{\alpha^2}{4} - \frac{\alpha^2}{8} = \frac{\beta^2}{4} + \frac{\alpha^2}{8}.
\end{aligned}$$

Now suppose  $S$  deviates and chooses  $x$  such that  $g(1-x) > \frac{1}{2} > g(x)$  and  $\beta a'_1 + g(x) \alpha \gamma'' = g(1-x) \alpha \gamma''$ , where now  $\gamma''$  is the level of investment chosen. (Such a  $x$  will clearly exist provided  $a'_1$  is not too high).

Given the choice of  $x$ ,  $\gamma''$  can be determined by the first-order conditions as before, so that

$$\gamma'' = g(1-x) \bullet \alpha.$$

The seller now gets the entire payoff because  $B_1$  and  $B_2$  bid away their entire surpluses.

Therefore the payoff to  $S$  is

$$\beta \bullet \frac{\beta}{2} + g(x) \bullet \alpha (1-x) \bullet \alpha - \frac{1}{2} (g(1-x) \alpha)^2.$$

Subtract from this  $S$ 's payoff in the candidate equilibrium, namely  $\frac{\beta^2}{4} + \frac{\alpha^2}{8}$ , to obtain

$$\frac{\beta^2}{4} + \alpha^2 \left[ g(x) \bullet g(1-x) - \frac{1}{2} (g(1-x))^2 - \frac{1}{8} \right].$$

But we can substitute now for  $\frac{\beta^2}{4}$ , since

$$\frac{\beta^2}{2} + g(x) \bullet \alpha \quad g(1-x) \alpha = (g(1-x))^2 \alpha^2.$$

The difference in payoffs is then

$$\begin{aligned}
&\alpha^2 \left[ \frac{(g(1-x))^2}{2} - \frac{g(x)g(1-x)}{2} + g(x)g(1-x) - \frac{(g(1-x))^2}{2} - \frac{1}{8} \right] \\
&= \frac{\alpha^2}{2} \left[ g(x)g(1-x) - \frac{1}{4} \right].
\end{aligned}$$

We know that when  $x = 0$ ,  $g(x) \bullet g(1-x) = 0$  and likewise for  $x = 1$ . Symmetry (and the concavity of the product) demonstrate that  $g(x)g(1-x)$

reaches its maximum value at  $x = \frac{1}{2}$ , but since each of the components of the product is greater than  $\frac{1}{2}$ , the product is greater than  $\frac{1}{4}$ .

Therefore, there could exist values of parameters such that  $g(x)g(1-x) > \frac{1}{4}$ , hence a profitable deviation could exist. In this case the candidate equilibrium is not actually an equilibrium.

We have considered only the asymmetric pure strategy profile with  $S$  investing at  $x = 0$  or  $x = 1$ . If  $S$  optimally invests so as to remove the asymmetry completely, the buyer who invests a positive amount will get a negative payoff and will deviate to 0 investment. The remaining case is the seller partially removing the asymmetry so as to get more than  $1/2$  the payoff with a particular buyer without completely removing it. (Given the condition in Section 4.2 of the relative unimportance of the buyer's investment compared to the seller's, the seller has an incentive to choose  $x$  strictly in the interior to induce a larger outside option (as in the proof of Proposition 1.) Such an asymmetric equilibrium would have the buyer investing optimally but the seller choosing  $x$  not equal to 0 or 1, thereby introducing inefficient investment-type choice.

## 5 Sequential Offers Extensive Form

### 5.1 The Bargaining Procedure

We now consider a bargaining procedure where a randomly chosen buyer,  $B_1$  or  $B_2$ , makes a proposal to the seller  $S$ . The seller can accept or reject. If the seller accepts, the game is over. If  $S$  rejects, she makes an offer, but chooses either one of the buyers to bargain with. The chosen buyer  $B_i$  then accepts or rejects and so on. Between a rejection and a new proposal, time elapses (as in Rubinstein (1982)) and the common discount factor is  $\delta$ , as before. The payoffs have already been described in the model section.

This extensive form has been studied, for example in Osborne and Rubin-

stein (1990) or in Chatterjee, Dutta, Ray and Sengupta (1993) where it is an example in a general study of coalition formation. We summarize the result in a proposition.

**Proposition 3** *Let  $v_1$  be the surplus if a  $B_1 - S$  trade takes place and  $v_2$  be the surplus if a  $B_2 - S$  trade takes place. Let  $(v_i - p, p)$  be a (feasible and efficient) agreement between  $B_i$  and  $S$  where the second entry is the payment from the buyer to the seller while the first entry is the part of surplus left over to the buyer,  $i = 1, 2$ . The subgame perfect equilibrium is as follows:*

1. *Suppose  $v_1 > v_2$  (the case of  $v_2 < v_1$  is symmetric and omitted).  $B_1$  always offers  $\left(\frac{1}{1+\delta}v_1, \frac{\delta}{1+\delta}v_1\right)$  and accepts offers that give him at least  $\frac{\delta}{1+\delta}v_1$  and rejects otherwise.  $B_2$  always offers  $\left(v_2 - \frac{\delta}{1+\delta}v_1, \frac{\delta}{1+\delta}v_1\right)$  if  $v_2 > \frac{\delta}{1+\delta}v_1$  and always makes a rejected offer otherwise.  $B_2$  always accepts offers that give him at least  $\delta \max\left(v_2 - \frac{\delta}{1+\delta}v_1, 0\right)$ .  $S$  always offers to  $B_1$ , with proposal  $\left(\frac{\delta}{1+\delta}v_1, \frac{1}{1+\delta}v_1\right)$ , and always accepts offers that give her at least  $\frac{\delta}{1+\delta}v_1$ , rejecting otherwise.*
2. *Suppose  $v_1 = v_2 = v$ , the two buyers always offer  $\left(\frac{1}{1+\delta}v, \frac{\delta}{1+\delta}v\right)$  and accept any offer that gives him at least  $\frac{\delta}{1+\delta}v$ . No player (buyers or seller) accepts an offer giving him or her less than  $\frac{\delta}{1+\delta}v$ .*

**Proof.** See Osborne and Rubinstein (1990) and Chatterjee et al. (1993). ■

The striking property of sequential offers is that the seller is not able to reap the benefits of being on the short side of the market. In the “auction-like” mechanism, the bargaining equilibrium is in the core of the game in that the seller obtains the entire surplus. These obviously correspond to different institutions. The “auction-like” mechanism appears to be more appropriate in a “bazaar” setting where buyers make simultaneous offers, while sequential offers

seems a better approximation (though by no means a perfect one) to bargains as diverse as those of house sales (in the US) to merger negotiations.

Paradoxically, the sequential offers bargaining might contain the appropriate incentives for investment, more than the auction mechanism described in the previous section. We now turn to this issue.

## 5.2 Seller Investment under Sequential Offers Bargaining

It is clear that there is no incentive for  $S$  under this bargaining procedure to choose any value of  $x$  other than 0 or 1.

**Proposition 4** *If the bargaining procedure used is sequential offers,  $S$  will set  $x = 0$  or 1 and choose*

$$c'(\gamma') = \frac{\delta}{1 + \delta} \alpha.$$

**Proof.** The payoff to the seller  $S$  is

$$\frac{\delta}{1 + \delta} \max_{x, \gamma} \{v_1, v_2\} - c(\gamma).$$

Since what matters is the maximum of  $v_1$  and  $v_2$ , the value of  $x$  must be either 0 or 1; any other value of  $x$  will multiply  $\alpha$  by a factor of  $g(x)$  or  $g(1 - x) < 1$ . Given this, the optimal value of  $\gamma$  is as stated in the proposition, from the first-order conditions, which are also sufficient. ■

Therefore, in Examples 1 and 2, where the seller investment decision had  $x \in (0, 1)$ , the sequential offers bargaining procedure will generate a larger total surplus.

## 5.3 Buyer Investment

We now consider buyer investment in this model. Since one of the buyers is randomly chosen to propose, there is a chance that the buyer for whom the seller has invested is not the first one to propose.

The expected payoff (excluding investment) to  $B_i$ , where  $i$  is the buyer chosen by the seller is therefore

$$\frac{1}{2} \left( \frac{1}{1+\delta} v_i(\gamma) \right) + \frac{1}{2} \frac{\delta^2}{1+\delta} v_i(\gamma).$$

The other buyer has an expected payoff of 0 and therefore will not invest.

For simplicity, suppose  $i = 1$ .<sup>7</sup>

**Proposition 5**  $B_1$  will invest an amount  $a'_1$  such that

$$\frac{1+\delta^2}{2} \frac{1}{1+\delta} \beta = c'(a'_1),$$

and  $B_2$  will invest 0.

**Proof.** Similar to previous propositions. ■

Note that in comparison to the “auction-like” mechanism, there is no inefficient over-investment by buyers, such as  $B_2$  investing a positive amount. (If an asymmetric equilibrium exists in the auction mechanism, and under the assumption that  $S$  prefers  $B_1$ , other things being equal, then this too avoids inefficient overinvestment and elicits more investment from  $B_1$  than under sequential offers (though the difference goes to 0 as  $\delta \rightarrow 1$ ).)

## 6 Discussion about property rights and conclusions

To summarise, this paper has considered three types of inefficiency in the choice of investment in a market setting. The first, well-known as the “hold-up problem”, involves a player investing too little because of the surplus division in the ex post bargaining. The second, present in our earlier paper and in Cai (2003), is about the type of investment chosen by the seller in order to exploit her

<sup>7</sup> $S$  has lexicographic preference. Other things being equal, she prefers  $B_1$  to  $B_2$ . (This is another way to state the assumption that  $i = 1$ .)

market power-generalist or specific. A third, minor, cause of inefficient buyer investment in this paper has to do with the order in which buyers approach the seller. While the first source of inefficiency can be reduced in the usual Hart-Moore way of giving property rights to the seller, mitigating the second cause would require property rights for the seller's asset to be held by one of the buyers (not jointly). These statements hold for the bargaining procedure that gives all the surplus to the short side of the market.

Bargaining and competition play important and interconnected roles in our paper. We do not take the view that all extensive forms can be chosen in all markets; the nature of the industry one is considering imposes constraints on the type of bargaining that can take place. With sequential offers, individual ownership of one's assets does as well as assignment of property rights in enhancing efficiency of investment. Thus, if a natural bargaining procedure is used arms length relationships can still be sufficient.

As pointed out earlier, the choice of specific versus general human capital investment appears quite frequently in the labour market. An electrical engineer who chooses to pursue an MBA instead of, say, a telecommunications degree, is choosing general investment over specific investment. Some would argue that this is, in fact, inefficient, though there are no evident property rights solutions for that problem. The scope of this paper is therefore broader than the incomplete contracts and property rights framework.

## References

- [1] Cai, Hongbin (2003), "A Theory of Joint Asset Ownership," *RAND Journal of Economics*, Vol.34, no. 1, Spring 2003.
- [2] Chatterjee, Kalyan and Y. Stephen Chiu, (2000), "When Does Competition Lead to Efficient Investments?" mimeo, School of Economics and Finance, University of Hong Kong.
- [3] Chatterjee, Kalyan and Bhaskar Dutta, (1998), "Rubinstein Auctions: On Competition for Bargaining Partners," *Games and Economic Behavior*, 23, May 1998, pp. 119-145.
- [4] Chatterjee, Kalyan and Bhaskar Dutta, Debraj Ray, and Kunal Sengupta (1993), "A Noncooperative Theory of Coalitional Bargaining," *Review of Economic Studies*, 60(2): 463-77.
- [5] Chatterjee, Kalyan and Robert C. Marshall (2003), "A Model of Academic Tenure," mimeo Penn State Department of Economics.
- [6] Chiu, Y. Stephen (1998), "Noncooperative Bargaining, Hostages, and Optimal Asset Ownership," *American Economic Review*, 88(4): 882-901.
- [7] Cole, Hal, George Mailath and Andrew Postlewaite (2001a), "Efficient Non-Contractible Investments in Large Economies," *Journal of Economic Theory*, 101, 2001, pp. 333-373.
- [8] \_\_\_\_\_, \_\_\_\_\_ and \_\_\_\_\_ (2001b), "Efficient Non-Contractible Investments in a Finite Economy," *Advances in Theoretical Economics*, Vol. 1 [2001], No. 1, Article 2.

- [9] de Meza, David and Ben Lockwood (1998a), “Does Asset Ownership Always Motivate Managers? Outside Options and the Property Rights Theory of the Firm,” *Quarterly Journal of Economics*, 113(2): 361-86.
- [10] \_\_\_\_\_ and \_\_\_\_\_ (1998b), “Investment, Asset Ownership and Matching: the Property Rights Theory of the Theory in Market Equilibrium,” mimeo, Universities of Exeter and Warwick.
- [11] Felli, Leonardo and Kevin W.S. Roberts, (2002), “Does Competition Solve the Hold-up Problem?” mimeo, LSE and Nuffield College, Oxford.
- [12] Gans, Joshua, (2003) “Markets for Ownership,” Melbourne Business School Working Paper.
- [13] Grossman, Sanford J. and Hart, Oliver D. (1986) “The Costs and Benefits of Ownership: A Theory of Vertical and Lateral Integration,” *Journal of Political Economy*, 94(4): 691-719.
- [14] Hart, Oliver and John Moore (1990), “Property Rights and the Nature of the Firm,” *Journal of Political Economy* 98: 1119-1158.
- [15] Muthoo, Abhinay, (1999), *Bargaining Theory with Applications*, Cambridge University Press, Cambridge and New York.
- [16] Osborne, Martin J. and Ariel Rubinstein, (1990), *Bargaining and Markets*, Academic Press, New York.
- [17] Rajan, Raghuram G. and Luigi Zingales (1998), “Power in a Theory of the Firm,” *Quarterly Journal of Economics*, 113(2): 387-432.
- [18] Rubinstein, Ariel, (1982) “Perfect Equilibrium in a Bargaining Mode,” *Econometrica*, Vol. 50,

- [19] Shaked, Avner (1994), "Opting out: bazaars versus 'hi tech' markets,"  
*Investigaciones Economicas*, 1994, vol. 18, issue 3, pages 421-432